Evaluate $\int \sec^6 x \tan^4 x \, dx$. $= \int \sec^4 x \tan^4 x \sec^2 x \, dx$

SCORE: _____ / 5 PTS

$\underline{u = \tan x}$ ①

$du = \sec^2 x \, dx$

$= \underline{\int (u^2+1)^2 u^4 \, du}$ ①

$= \int \underline{(u^8 + 2u^6 + u^4)} \, du$ ①

$= \underline{\frac{1}{9}u^9 + \frac{2}{7}u^7 + \frac{1}{5}u^5} + C$ ①

$= \underline{\frac{1}{9}\tan^9 x + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x} + C$ ①

$\left(-\frac{1}{2}\right)$ IF YOU FORGOT +C AT THE END

Evaluate $\int (5x^2 - x + 3)\sin 2x \, dx$. $= \underbrace{\left(-\frac{5}{2}x^2 + \frac{1}{2}x - \frac{3}{2}\right)\cos 2x}$ ②

SCORE: ____ / 5 PTS

$\left(1\frac{1}{2}\right)$ $\underbrace{+\left(\frac{5}{2}x - \frac{1}{4}\right)\sin 2x} + \underbrace{\frac{5}{4}\cos 2x}$ ① $+ C$

u		dv
$5x^2 - x + 3$	$+$	$\sin 2x$
$10x - 1$	$-$	$-\frac{1}{2}\cos 2x$
10	$+$	$-\frac{1}{4}\sin 2x$
0		$\frac{1}{8}\cos 2x$

$$= \underbrace{\left(-\frac{5}{2}x^2 + \frac{1}{2}x - \frac{1}{4}\right)\cos 2x + \left(\frac{5}{2}x - \frac{1}{4}\right)\sin 2x}_{\left(\frac{1}{2}\right)} + C$$

$\left(-\frac{1}{2}\right)$ IF YOU FORGOT $+C$

Evaluate $\int e^{-2x}\cos 5x\,dx$. $= -\frac{1}{2}e^{-2x}\cos 5x + \frac{5}{4}e^{-2x}\sin 5x$ ② **SCORE: _____ / 5 PTS**

$-\frac{25}{4}\int e^{-2x}\cos 5x\,dx$ ①

u		dv
$\cos 5x$	$+$	e^{-2x}
$-5\sin 5x$	$-$	$-\frac{1}{2}e^{-2x}$
$-25\cos 5x$	$+$	$\frac{1}{4}e^{-2x}$

$$\frac{29}{4}\int e^{-2x}\cos 5x\,dx = -\frac{1}{2}e^{-2x}\cos 5x + \frac{5}{4}e^{-2x}\sin 5x$$ ①

$$\int e^{-2x}\cos 5x\,dx = -\frac{2}{29}e^{-2x}\cos 5x + \frac{5}{29}e^{-2x}\sin 5x + C$$

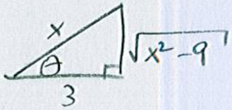
①

($-\frac{1}{2}$) IF YOU FORGOT $+C$

Evaluate $\int (x^2-9)^{\frac{3}{2}}\,dx$.

SCORE: _____ / 9 PTS

$x = 3\sec\theta$ ① $\longrightarrow$ $\sec\theta = \frac{x}{3}$



$dx = 3\sec\theta\tan\theta\,d\theta$

$\int (9\sec^2\theta - 9)^{\frac{3}{2}} \cdot 3\sec\theta\tan\theta\,d\theta$

$= \int$ (½) $(9\tan^2\theta)^{\frac{3}{2}} \cdot 3\sec\theta\tan\theta\,d\theta$ ①

① $= 81\int \sec\theta\tan^4\theta\,d\theta$

$= 81\int \sec\theta(\sec^2\theta - 1)^2\,d\theta$

① $= 81\left[\int \sec^5\theta\,d\theta - 2\int\sec^3\theta\,d\theta + \int\sec\theta\,d\theta\right]$

$= 81\left[\right.$ ① $\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int\sec^3\theta\,d\theta - 2\int\sec^3\theta\,d\theta + \int\sec\theta\,d\theta\left.\right]$

$= 81\left[\frac{1}{4}\sec^3\theta\tan\theta - \frac{5}{4}\left(\frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|)\right)\right.$ ①

$\left. + \ln|\sec\theta + \tan\theta|\right]$ (½) $+ C$

$= 81\left[\frac{1}{4}\sec^3\theta\tan\theta - \frac{5}{8}\sec\theta\tan\theta + \frac{3}{8}\ln|\sec\theta + \tan\theta|\right] + C$ (½)

① $= 81\left[\frac{1}{4}\left(\frac{x}{3}\right)^3\frac{\sqrt{x^2-9}}{3} - \frac{5}{8}\,\frac{x}{3}\,\frac{\sqrt{x^2-9}}{3} + \frac{3}{8}\ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right|\right] + C$

(−½) IF YOU FORGOT +C AT THE END

$= \frac{1}{4}x^3\sqrt{x^2-9} - \frac{45}{8}x\sqrt{x^2-9} + \frac{243}{8}\ln\left|x + \sqrt{x^2-9}\right| + C$ (½)

Prove the reduction formula $\int \cos^n u\, du = \frac{1}{n}\cos^{n-2} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du$. **SCORE: _____ / 6 PTS**

NOTE: You must show how to get this formula.
You will receive 0 credit if your "proof" is differentiating both sides of the equation.

$$\begin{array}{ccc} \underline{f} & & \underline{g} \\ \cos^{n-1} u & \overset{+}{\searrow} & \cos u \\ -(n-1)\cos^{n-2} u \sin u & \leftarrow & \sin u \end{array}$$

$$\int \cos^n u\, du = \underline{\cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u \sin^2 u\, du} \;②$$

$$= \underline{\cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u\,(1-\cos^2 u)\, du} \;(1\tfrac{1}{2})$$

$$= \underline{\cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u\, du - (n-1)\int \cos^n u\, du} \;①$$

$$\underline{n\int \cos^n u\, du = \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u\, du} \;①$$

$$\underline{\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du} \;(\tfrac{1}{2})$$